Chapter 14

Name______________________________ Date__________

Chapter 14 Real Numbers and the Pythagorean Theorem

Dear Family,

When adding or multiplying small numbers, you rely on tables you memorized long ago. For larger numbers, you follow the rules you've learned. For example, when adding large numbers, you line up the place values and start adding from the right, carrying digits to the left.

The "add and carry" method is an example of a rule that follows a strict, predictable procedure. Perhaps surprisingly, not all problems in mathematics have rules that are this straightforward. One of the oldest ways of solving problems is to use the "guess and check" method.

This method requires us to make a reasonable guess about the answer and check how close it is. You then refine your guess and check the new estimate. Each time you do this, you try to get closer to the answer.

Try this with your student to find the square root of a number. For example, to find the square root of 19, you might do the following steps.

- The square root of 16 is 4 $\left(\text{because } 4^2 = 16\right)$ and the square root of 25 is 5 $\left(\text{because } 5^2 = 25\right)$. Because 19 is between 16 and 25, the square root of 19 is greater than 4 and less than 5, so guess 4.5.
- Check: $(4.5)^2 = 20.25$, which is too big, so refine your guess. Try 4.2.
- Check: $(4.2)^2 = 17.64$, which is too small, so refine your guess. Try 4.4.
- Check: $(4.4)^2 = 19.36$, which is getting closer, but still a little too big.

If you continue this method, you will soon find out that $19 \approx (4.36)^2$. You could keep going to get the precision you need.

It may appear that computers and calculators have functions like these memorized, because the answers are shown immediately. However, many types of calculations are done using a process very similar to "guess and check." Because computers and calculators can make millions of guesses per second, the answer simply appears to be memorized.

So don't be afraid to guess the answer—just remember to check it!

Nombre ____ Fecha ________

Capítulo 14 Números reales y el Teorema Pitagórico

Estimada Familia:

Al sumar o multiplicar números pequeños, dependemos de tablas que memorizamos hace muchos años. Para números más grandes, seguimos reglas que hemos aprendido. Por ejemplo, al sumar números grandes, alineamos las posiciones de valores y empezamos a sumar desde el lado derecho, llevando dígitos hacia el lado izquierdo.

El método de "sumar y llevar" es un ejemplo de una regla que sigue un procedimiento estricto y predecible. Quizás, y sorprendentemente, no todos los problemas en matemáticas tienen reglas tan simples como ésta. Una de las formas más antiguas de resolver problemas es usando el método de "predecir y verificar".

Este método requiere que hagamos una predicción razonable sobre la respuesta y que verifiquemos qué tan cerca estamos. Luego refinamos la predicción y verificamos la nueva aproximación. Cada vez que hacemos esto, estamos más cerca de la respuesta.

Intente esto con su estudiante para hallar la raíz cuadrada de un número. Por ejemplo, para encontrar la raíz cuadrada de 19, pueden hacer los siguientes pasos:

- La raíz cuadrada de 16 es 4 $\left(\text{porque } 4^2 = 16\right)$ y la raíz cuadrada de 25 es 5 $\left(\text{porque } 5^2 = 25\right)$. Ya que 19 se encuentra entre 16 y 25, la raíz cuadrada de 19 es mayor que 4 y menor que 5, entonces predecimos 4.5.
- Verifique: $(4.5)^2 = 20.25$, que es demasiado grande, así que refine su predicción. Intente con 4.2.
- Verificar: $(4.2)^2 = 17.64$, que es demasiado pequeño, así que refine su predicción. Intente con 4.4.
- Verificar: $(4.4)^2 = 19.36$, lo cual está más cerca, pero todavía es un poco más grande.

Si continúa con este método, pronto averiguará que $19 \approx (4.36)^2$. Puede continuar para obtener la precisión deseada.

Puede parecer que las computadoras y calculadoras tengan funciones como éstas memorizadas, ya que las respuestas se muestran inmediatamente. Sin embargo, muchos tipos de cálculos se realizan con un proceso muy similar al de "predecir y verificar." Ya que las computadoras y calculadoras pueden hacer millones de predicciones por segundo, la respuesta simplemente aparece como memorizada.

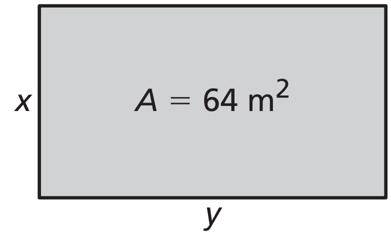
Así que no tema predecir la respuesta—¡sólo recuerde verificarla!

Activity 14.1

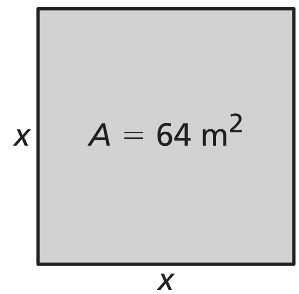
Start Thinking!

For use before Activity 14.1

When you know the area of a rectangle, can you determine the lengths of its sides? Why or why not?



When you know the area of a square, can you determine the lengths of its sides? Why or why not?



Activity 14.1

Warm Up

For use before Activity 14.1

Find the product.

1. 12×12 **2.** 9×9 **3.** 18×18

4. 1.6×1.6 **5.** 2.5×2.5 **6.** $\dfrac{2}{3} \times \dfrac{2}{3}$

Lesson 14.1

Start Thinking!

For use before Lesson 14.1

Shelley says that there are two solutions of the equation $x^2 = 400$. Gina says that there is only one solution. Who is correct? Explain.

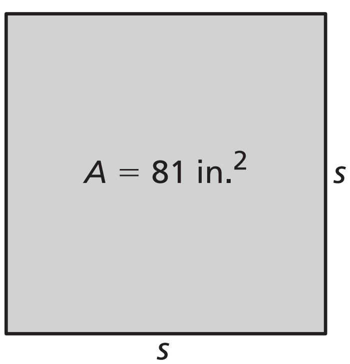
Lesson 14.1

Warm Up

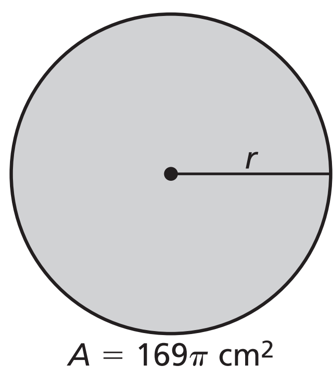
For use before Lesson 14.1

Find the dimensions of the square or circle. Check your answer.

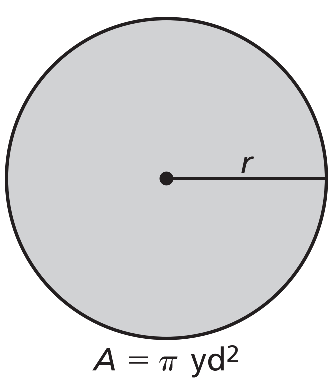
1.



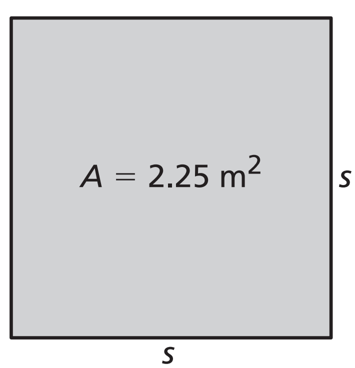
2.



3.



4.

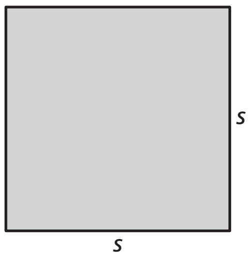


Name______________________________ Date__________

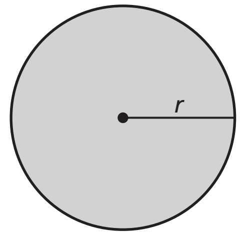
14.1 Practice A

Find the dimensions of the square or circle. Check your answer.

1. Area $= 196 \text{ in.}^2$



2. Area $= 36\pi \text{ m}^2$



Find the two square roots of the number.

3. 16

4. 0

Find the square root(s).

5. $\sqrt{121}$

6. $-\sqrt{\dfrac{1}{36}}$

7. $\pm\sqrt{\dfrac{289}{49}}$

8. $-\sqrt{0.64}$

Evaluate the expression.

9. $2\sqrt{25} + 3$

10. $7 - 12\sqrt{\dfrac{1}{9}}$

Copy and complete the statement with $<$, $>$, or $=$.

11. $\sqrt{64}$ __?__ 5

12. 0.6 __?__ $\sqrt{0.49}$

13. The volume of a right circular cylinder is represented by $V = \pi r^2 h$, where r is the radius of the base (in feet). What is the radius of a right circular cylinder when the volume is 144π cubic feet and the height is 9 feet?

14. The cost C (in dollars) of producing x widgets is represented by $C = 4.5x^2$. How many widgets are produced if the cost is \$544.50?

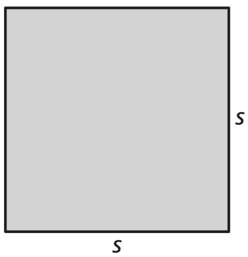
15. Two squares are drawn. The larger square has area of 400 square inches. The areas of the two squares have a ratio of 1 : 4. What is the side length s of the smaller square?

Name ____ Date _________

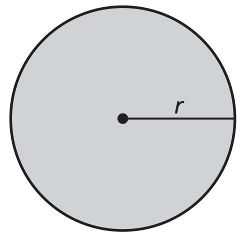
14.1 Practice B

Find the dimensions of the square or circle. Check your answer.

1. Area $= \dfrac{169}{225}$ cm^2



2. Area $= 121\pi$ yd^2



Find the two square roots of the number.

3. 225

4. 400

Find the square root(s).

5. $-\sqrt{484}$

6. $\pm\sqrt{\dfrac{25}{64}}$

7. $\sqrt{6.25}$

8. $\pm\sqrt{1.69}$

Evaluate the expression.

9. $6\sqrt{2.25} - 4.2$

10. $3\left(\sqrt{\dfrac{48}{3}} - 2\right)$

Copy and complete the statement with $<$, $>$, or $=$.

11. $\sqrt{\dfrac{49}{9}}$ __?__ 2

12. $\dfrac{2}{5}$ __?__ $\sqrt{\dfrac{12}{75}}$

13. The area of a sector of a circle is represented by $A = \dfrac{5}{18}\pi r^2$, where r is the radius of the circle (in meters). What is the radius when the area is 40π square meters?

14. Two squares are drawn. The smaller square has an area of 256 square meters. The areas of the two squares have a ratio of 4 : 9. What is the side length s of the larger square?

Name______________________________ Date__________

14.1 Enrichment and Extension

Solving Equations with Square Roots

A variable is used to represent an unknown number. Variables also have square roots and can be used to solve equations. Use inverse operations prior to taking the square root to solve the equation.

Example: Solve $2x^2 - 4 = 4$.

$2x^2 - 4 = 4$	Write the equation.
$2x^2 = 8$	Add 4 to each side.
$x^2 = 4$	Divide each side by 2.
$x = \pm 2$	Find both square roots of 4.

Solve the equation using square roots.

1. $p^2 - 49 = 0$

2. $a^2 - 100 = 0$

3. $r^2 - 16 = 0$

4. $8 = j^2 + 4$

5. $d^2 - 12 = 4$

6. $y^2 + 2 = 6$

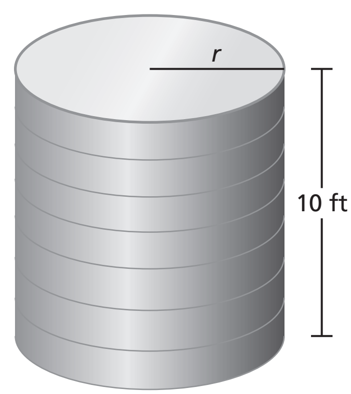
7. $4x^2 - 81 = 19$

8. $s^2 - 2 = -1$

9. $5t^2 + 2 = 127$

10. $-p^2 - 15 = -24$

11. An oil tank holds about 785 cubic feet of oil and has a height of 10 feet. The formula for the volume of a cylinder is $V = \pi r^2 h$. Find the radius of the tank. Use 3.14 for π.



Name ____ Date __________

14.1 Puzzle Time

How Did The Man At The Seafood Restaurant Cut His Mouth?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

Find the two square roots of the number.

1. 169

2. 576

3. $\frac{49}{64}$

4. 2.56

Find the square root(s).

5. $\sqrt{400}$

6. $-\sqrt{225}$

7. $\pm\sqrt{\frac{9}{16}}$

8. $\sqrt{\frac{36}{25}}$

9. $\pm\sqrt{7.84}$

10. $-\sqrt{56.25}$

Evaluate the expression.

11. $6 - 2\sqrt{81}$

12. $\sqrt{53.29} + \sqrt{2.89}$

13. $\sqrt{21.16} - \sqrt{1.69}$

14. $7\sqrt{\frac{25}{49}} + \sqrt{\frac{36}{64}}$

15. The bottom of a circular swimming pool has an area of 200.96 square feet. What is the radius (in feet) of the swimming pool? Use 3.14 for π.

R	E	L	C	A	F	T	M	I	H	N	U	S	B	G	R	D
25	±2.8	−10	7.5	±1.6	2.3	$\pm\frac{3}{4}$	$4\frac{3}{4}$	±13	28	$\pm\frac{7}{8}$	±3.4	$4\frac{1}{3}$	−5.5	−12	30	±5.2
S	**I**	**T**	**W**	**N**	**O**	**P**	**R**	**G**	**D**	**V**	**F**	**I**	**Y**	**S**	**L**	**H**
−15	$3\frac{1}{4}$	−6.5	$5\frac{3}{4}$	3.4	20	±1.8	$\frac{6}{5}$	12	8	−1.6	3.3	±24	−6.1	−7.5	14	9

Activity 14.2 Start Thinking!
For use before Activity 14.2

Your friend says that $\sqrt{169}$ is 13. Another friend says that $\sqrt{169}$ is -13. Who is correct? Explain.

Activity 14.2 Warm Up
For use before Activity 14.2

Find the square root(s).

1. $\sqrt{36}$

2. $-\sqrt{64}$

3. $\sqrt{\dfrac{49}{81}}$

4. $-\sqrt{225}$

5. $\sqrt{121}$

6. $\sqrt{\dfrac{144}{169}}$

Lesson 14.2

Start Thinking!

For use before Lesson 14.2

Explain to a partner what the difference is between finding the square root of a number and finding the cube root of a number. Use an example to support your reasoning.

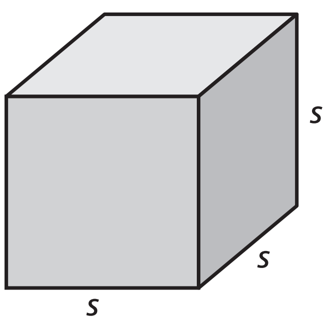
Lesson 14.2

Warm Up

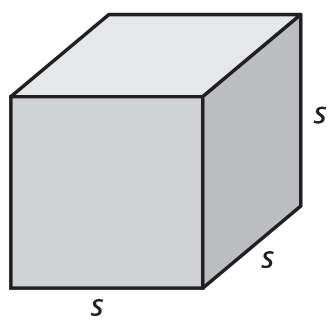
For use before Lesson 14.2

Find the edge length of the cube.

1. Volume $= 64{,}000 \text{ ft}^3$



2. Volume $= \dfrac{1}{216} \text{ ft}^3$

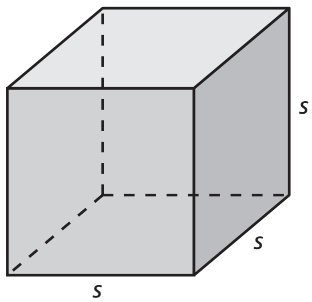


Name____ Date__________

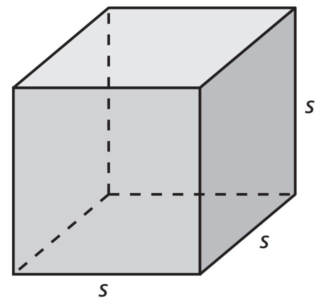
14.2 Practice A

Find the edge length of the cube.

1. Volume $= 27{,}000 \text{ cm}^3$



2. Volume $= \frac{1}{8} \text{ in.}^3$



Find the cube root.

3. $\sqrt[3]{125}$

4. $\sqrt[3]{-1}$

5. $\sqrt[3]{-8}$

6. $\sqrt[3]{-1000}$

7. $\sqrt[3]{8000}$

8. $\sqrt[3]{512}$

9. $\sqrt[3]{-\frac{1}{64}}$

10. $\sqrt[3]{0.001}$

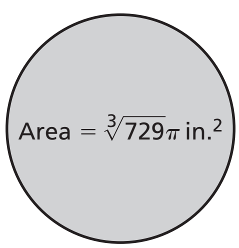
Copy and complete the statement with <, >, or =.

11. $-\sqrt[3]{27}$ __?__ -4

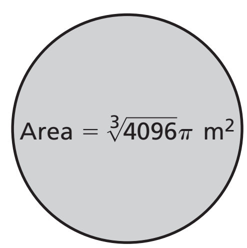
12. $\sqrt[3]{64}$ __?__ $\sqrt{16}$

Find the circumference of the circle.

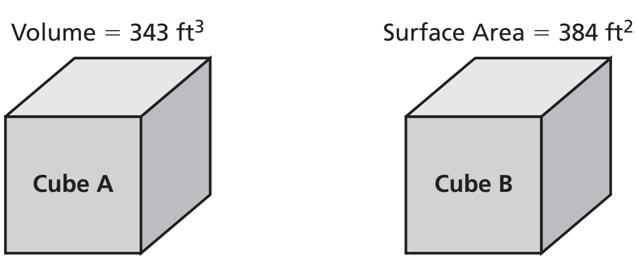
13.



14.



15. Which cube has a greater edge length? How much greater is it?



Name ____ Date __________

14.2 Practice B

Find the cube root.

1. $\sqrt[3]{343}$ **2.** $\sqrt[3]{-1331}$ **3.** $\sqrt[3]{-8000}$

4. $\sqrt[3]{3375}$ **5.** $\sqrt[3]{\dfrac{1}{64}}$ **6.** $\sqrt[3]{-\dfrac{125}{27}}$

Evaluate the expression.

7. $13 + \left(\sqrt[3]{125}\right)^3$ **8.** $2\dfrac{2}{3} - \left(\sqrt[3]{\dfrac{1}{27}}\right)^3$ **9.** $24 + \left(\sqrt[3]{-1000}\right)^3$

Evaluate the expression for the given value of the variable.

10. $\sqrt[3]{4t} + 3t,\ t = 54$ **11.** $\sqrt[3]{\dfrac{n}{24}} - \dfrac{n}{25},\ n = 375$

12. The volume of storage pod that is shaped like a cube is 1728 cubic feet.

- **a.** What is the edge length of the storage pod?
- **b.** What is the surface area of the storage pod?
- **c.** What is the area of the floor space of the storage pod?

Copy and complete the statement with $<$, $>$, or $=$.

13. 0.25 __?__ $\sqrt[3]{0.008}$ **14.** $\sqrt{729}$ __?__ $\sqrt[3]{729}$

15. There are infinitely many pairs of numbers of which the sum of their cube roots is zero. Give two of these pairs.

16. The radius of a sphere can be represented by $r = \sqrt[3]{\dfrac{3V}{4\pi}}$, where V is the volume of the sphere. What is the radius of a sphere with a volume of 36π cubic meters?

Solve the equation.

17. $(4x - 1)^3 = 343$ **18.** $(15x^3 - 2)^3 = 2197$

Name______________________________ Date__________

14.2 Enrichment and Extension

Finding *n*th Roots

A square root of a number is a number that, when multiplied by itself, equals the given number. A cube root of a number is a number that, when used as a factor in a product three times, equals the given number. The *n*th root of a number x is a number r that, when used as a factor in a product n times, equals the given number x. The notation for the nth root of x is $\sqrt[n]{x}$.

$$r \bullet r \bullet r \bullet \ldots \bullet r = r^n = x \qquad \sqrt[n]{x} = r$$

1. Complete the table.

r	n	$x = r^n$	$\sqrt[n]{x}$	**Check**
1	7	1	1	$1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1$
2	6			
3	5			
4	4			

Simplify the expression.

2. $\sqrt[5]{32}$ **3.** $\sqrt[4]{81}$ **4.** $\sqrt[4]{625}$ **5.** $\sqrt[6]{729}$

6. Find the least whole number by which 9000 can be multiplied so that the product is a perfect cube. Explain your method.

7. Find the least whole number by which 3072 can be divided so that the quotient is a perfect cube. Explain your method.

Name ____ Date __________

14.2 Puzzle Time

What Kind of Coat Is Made Without Buttons?

Write the letter of each answer in the box containing the exercise number.

Find the cube root.

1. $\sqrt[3]{216}$

2. $\sqrt[3]{-343}$

3. $\sqrt[3]{\dfrac{27}{64}}$

4. $\sqrt[3]{-\dfrac{1000}{1331}}$

Evaluate the expression.

5. $21 - \sqrt[3]{729}$

6. $\sqrt[3]{-\dfrac{1}{125}} + 6\dfrac{1}{2}$

7. $6\sqrt[3]{-512} + 13$

8. $\left(\sqrt[3]{-2744}\right)^3 + 2800$

Evaluate the expression for the given value of the variable.

9. $3a - \sqrt[3]{5a}, a = 25$

10. $\sqrt[3]{-\dfrac{x}{5}} + \dfrac{x}{10}, x = 320$

11. The volume of a box is 8000 cubic millimeters. What is the edge length of the box?

Answers

A. $-\dfrac{10}{11}$

I. 6

T. 56

A. $\dfrac{3}{4}$

T. -35

C. 20

F. 12

N. 70

O. -7

P. 28

O. $6\dfrac{3}{10}$

11	6	3	8		2	5		10	4	1	9	7

Activity 14.3 Start Thinking!
For use before Activity 14.3

Cut three narrow strips of paper that are 3 inches, 4 inches, and 5 inches long.

Form a triangle using the three strips. What kind of a triangle is formed?

Notice that $3^2 + 4^2 = 5^2$.

Do you know any other triangles whose side lengths would satisfy a similar equation?

Activity 14.3 Warm Up
For use before Activity 14.3

Find the square root(s).

1. $\sqrt{1.44}$ **2.** $\pm\sqrt{900}$ **3.** $\sqrt{\frac{4}{9}}$

4. $-\sqrt{441}$ **5.** $\pm\sqrt{484}$ **6.** $-\sqrt{2500}$

Lesson 14.3

Start Thinking!

For use before Lesson 14.3

How can you use the Pythagorean Theorem in sports?

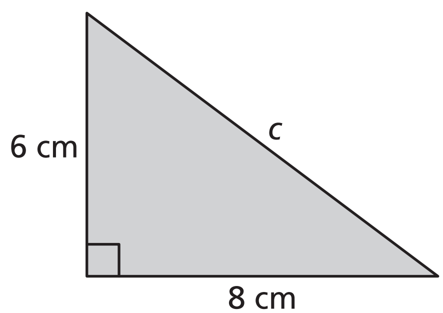
Lesson 14.3

Warm Up

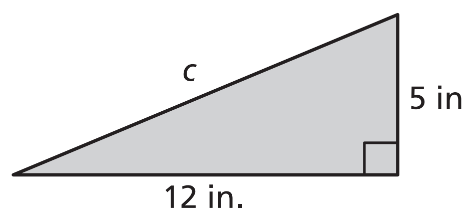
For use before Lesson 14.3

Find the missing length of the triangle.

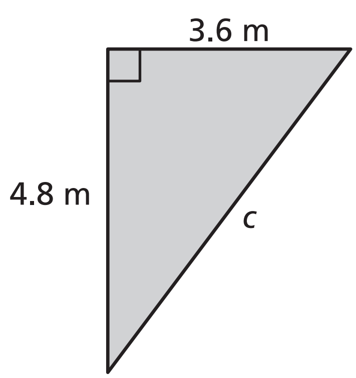
1.



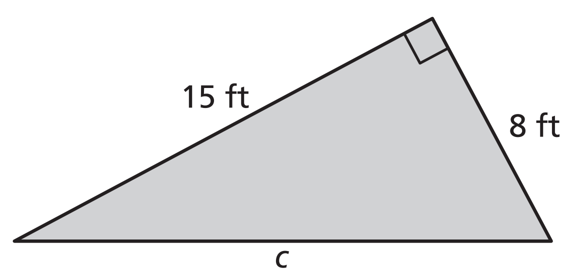
2.



3.



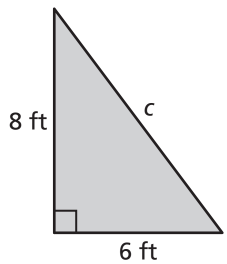
4.

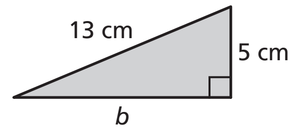


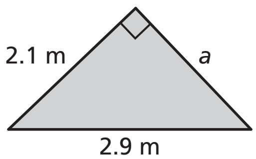
Name________________________________ Date__________

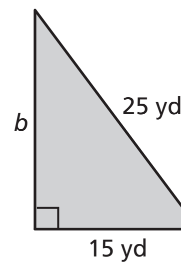
14.3 Practice A

Find the missing length of the triangle.

1. 

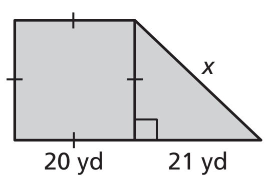
2. 

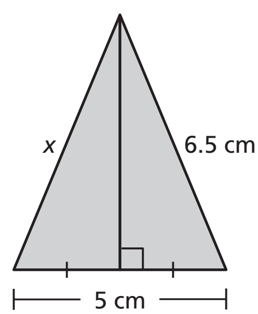
3. 

4. 

5. A small shelf sits on two braces that are in the shape of a right triangle. The leg (brace) attached to the wall is 4.5 inches and the hypotenuse is 7.5 inches. The leg holding the shelf is the same length as the width of the shelf. What is the width of the shelf?

Find the missing length of the figure.

6. 

7. 

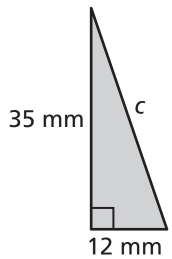
8. Can a right triangle have a leg that is 10 meters long and a hypotenuse that is 10 meters long? Explain.

9. One leg of a right triangular piece of land has a length of 24 yards. The hypotenuse has a length of 74 yards. The other leg has a length of $10x$ yards. What is the value of x?

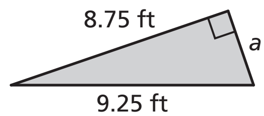
14.3 Practice B

Find the missing length of the triangle.

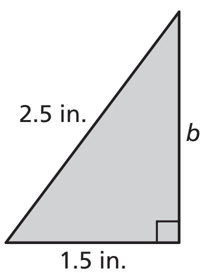
1.



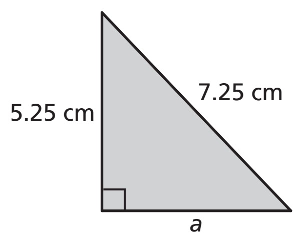
2.



3.



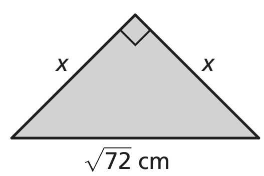
4.



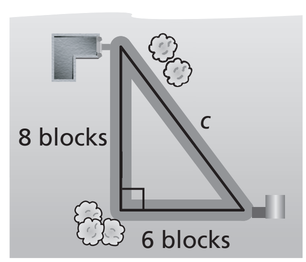
5. You built braces in the shape of a right triangle to hold your surfboard. The leg (brace) attached to the wall is 10 inches and your surfboard sits on a leg that is 24 inches. What is the length of the hypotenuse that completes the right triangle?

6. Laptops are advertised by the lengths of the diagonals of the screen. You purchase a 15-inch laptop and the width of the screen is 12 inches. What is the height of its screen?

7. In a right isosceles triangle, the lengths of both legs are equal. For the given isosceles triangle, what is the value of x?



8. To get from your house to your school, you ride your bicycle 6 blocks west and 8 blocks north. A new road is being built that will go directly from your house to your school, creating a right triangle. When you take the new road to school, how many fewer blocks will you be riding to school and back?

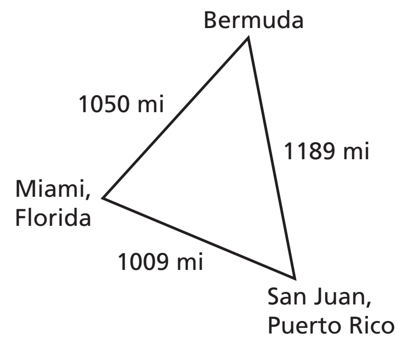


Name______________________________________ Date__________

14.3 Enrichment and Extension

The Bermuda Triangle

The Bermuda Triangle is in the Atlantic Ocean between Bermuda, Miami, Florida, and San Juan, Puerto Rico. There are many stories about strange events that occur within the Bermuda Triangle.



The Bermuda Triangle is not a right triangle. In order to find the area, you need to use a different method.

1. Find the perimeter of the triangle.

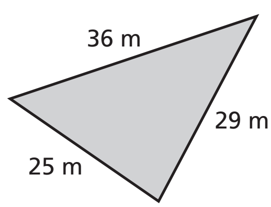
2. The semi-perimeter of a triangle is equal to half the perimeter. Find the semi-perimeter s of the triangle.

3. Find the differences between the semi-perimeter and each side of the triangle, $s - a$, $s - b$, and $s - c$.

4. Use the values you found to evaluate the product $R = s(s - a)(s - b)(s - c)$.

5. The area of the triangle is equal to $\sqrt{R}$. What is the area (in square miles) of the Bermuda Triangle?

6. This method of finding the area of a triangle is called Heron's Formula. Use this method to find the area of the triangle below.



Name ____ Date __________

14.3 Puzzle Time

What Did One Dog Say To The Other Dog?

Write the letter of each answer in the box containing the exercise number.

Find the hypotenuse *c* of the right triangle with the given side lengths *a* and *b*.

1. $a = 15, b = 20$

2. $a = 5, b = 12$

3. $a = 13, b = 84$

4. $a = 65, b = 72$

5. $a = 6, b = 17.5$

6. $a = 6\frac{2}{3}, b = 7$

Find the side length *b* of the right triangle with the given hypotenuse *c* and side length *a*.

7. $c = 61, a = 11$

8. $c = 82, a = 80$

9. $c = 34, a = 16$

10. $c = 65, a = 63$

11. $c = 13, a = 6.6$

12. $c = 10\frac{3}{5}, a = 5\frac{3}{5}$

13. The flap of an envelope has two side lengths that are each 10 centimeters long and meet at a right angle. How long is the envelope? Round your answer to the nearest tenth.

14. A middle school gym is 60 feet wide and 100 feet long. If you stand in one corner of the gym, how many feet away is the corner diagonally across from you? Round your answer to the nearest tenth.

Answers

T. $9\frac{2}{3}$

P. 14.1

E. 18.5

D. 18

N. 25

U. 9

O. 97

H. 116.6

N. 60

G. 30

O. 13

M. 11.2

I. 85

S. 16

10	6	2	13		14	4	12	1	8	3	7	9		11	5

Activity 14.4 Start Thinking!
For use before Activity 14.4

An irrational number is a number that cannot be written as a ratio of integers. Decimals that do not repeat and do not terminate are irrational.

Do you know any examples of irrational numbers?

Activity 14.4 Warm Up
For use before Activity 14.4

Use the Pythagorean Theorem to find the hypotenuse of a right triangle with the given leg lengths.

1. 30, 40

2. 10, 24

3. 16, 30

4. 9, 40

5. 54, 72

6. 2.5, 6

Lesson 14.4 Start Thinking!

For use before Lesson 14.4

How can you find the side length of a square that has the same area as an 8.5-inch-by-11-inch piece of paper?

Lesson 14.4 Warm Up

For use before Lesson 14.4

Tell whether the rational number is a reasonable approximation of the square root.

1. $\frac{577}{408}, \sqrt{2}$

2. $\frac{401}{110}, \sqrt{8}$

3. $\frac{271}{330}, \sqrt{21}$

4. $\frac{521}{233}, \sqrt{5}$

5. $\frac{795}{153}, \sqrt{27}$

6. $\frac{441}{150}, \sqrt{12}$

Name____ Date__________

14.4 Practice A

Tell whether the rational number is a reasonable approximation of the square root.

1. $\frac{277}{160}, \sqrt{3}$

2. $\frac{590}{160}, \sqrt{17}$

Classify the real number.

3. $-\sqrt{14}$

4. $1.\overline{3}$

5. 2.375

6. $\sqrt{100}$

7. You are finding the area of a circle with a radius of 2 feet. Is the area a *rational* or *irrational* number? Explain.

Estimate the square root to the nearest (a) integer and (b) tenth.

8. $\sqrt{33}$

9. $\sqrt{630}$

10. $-\sqrt{8}$

11. $\sqrt{\frac{7}{2}}$

12. A swimming pool is in the shape of a right triangle. One leg has a length of 10 feet and one leg has a length of 15 feet. Estimate the length of the hypotenuse to the nearest integer.

Which number is greater? Explain.

13. $\sqrt{70}, 8$

14. $-\sqrt{16}, 3$

15. $\sqrt{210}, 16\frac{1}{4}$

16. $\sqrt{\frac{4}{25}}, \frac{3}{10}$

17. Find a number a such that $2 < \sqrt{a} < 3$.

18. Is $\sqrt{\frac{1}{9}}$ a rational number? Explain.

19. Is $\sqrt{\frac{5}{9}}$ a rational number? Explain.

20. Is $\sqrt{\frac{2}{18}}$ a rational number? Explain.

Name ____ Date __________

14.4 Practice B

Tell whether the rational number is a reasonable approximation of the square root.

1. $\frac{2999}{490}, \sqrt{41}$

2. $\frac{2298}{490}, \sqrt{22}$

Classify the real number.

3. $2\frac{2}{9}$

4. $-\sqrt{576}$

5. $2.\overline{41}$

6. $\sqrt{130}$

7. You are finding the circumference of a circle with a diameter of 10 meters. Is the circumference a *rational* or *irrational* number? Explain.

Estimate the square root to the nearest (a) integer and (b) tenth.

8. $-\sqrt{\frac{250}{9}}$

9. $\sqrt{395}$

10. $\sqrt{0.79}$

11. $\sqrt{1.48}$

12. A patio is in the shape of a square, with a side length of 35 feet. You wish to draw a black line down one diagonal.

a. Use the Pythagorean Theorem to find the length of the diagonal. Write your answer as a square root.

b. Find the two perfect squares that the length of the diagonal falls between.

c. Estimate the length of the diagonal to the nearest tenth.

Which number is greater? Explain.

13. $\sqrt{220}, 14\frac{3}{4}$

14. $-\sqrt{135}, -\sqrt{145}$

15. $\sqrt{\frac{7}{64}}, \frac{3}{8}$

16. $-0.25, -\sqrt{\frac{1}{4}}$

17. Find two numbers a and b such that $7 < \sqrt{a} < \sqrt{b} < 8$.

Name____ Date__________

14.4 Enrichment and Extension

Approximating Square Roots

Before there were calculators and computers, mathematicians developed several methods of approximating square roots by hand. One popular method is sometimes called the divide-and-average method. It uses the following steps.

Use the divide-and-average method to calculate $\sqrt{47}$.

1. What two perfect squares is 47 between?
2. Let $g = \sqrt{47}$. Estimate g to the nearest whole number.
3. Find the quotient $q = 47 \div g$. Round your answer to two decimal places.
4. Find the average of g and q. This gives the approximate value of $\sqrt{47}$. To get a closer approximation, you can repeat this process multiple times by using the average as g.
5. Check the accuracy by squaring the average and comparing it to 47. How close are the numbers?
6. Use this method to estimate $\sqrt{30}$ by repeating the process three times. How close is the square of the estimate to 30?

Name ______________________________ Date __________

14.4 Puzzle Time

Did You Hear About...

A	B	C	D	E	F
G	H	I	J	K	L
M					

Complete each exercise. Find the answer in the answer column. Write the word under the answer in the box containing the exercise letter.

18.5 BECAUSE	-9 IN
$\sqrt{\frac{4}{9}}$ AND	13 DUTY
7 BECAME	-18 SAND
-34 SEASHELL	-35 LOBSTER
6.8 POLICEMAN	$-\sqrt{0.85}$ CLAM
$\sqrt{55}$ OCEAN	2.3 HE
11.7 A	29.2 ORDER
14 THE	$-\sqrt{0.75}$ CLAW
$-\sqrt{83}$ COURT	$\frac{1}{2}$ LAWYER
12 BELIEVED	-17 THAT

Estimate to the nearest integer.

A. $\sqrt{195}$

B. $-\sqrt{1220}$

C. $-\sqrt{306}$

D. $\sqrt{\frac{315}{6}}$

Estimate to the nearest tenth.

E. $\sqrt{137}$

F. $\sqrt{45.9}$

G. $\sqrt{342.5}$

H. $\sqrt{\frac{38}{7}}$

Which number is greater?

I. $\sqrt{55}$, 12

J. $-\sqrt{83}$, -9

K. $-\sqrt{0.75}$, $-\sqrt{0.85}$

L. $\sqrt{\frac{4}{9}}$, $\frac{1}{2}$

M. You are standing 15 feet from a 25-foot-tall tree. Estimate the distance from where you are standing to the top of the tree? Round your answer to the nearest tenth.

Extension 14.4

Start Thinking!

For use before Extension 14.4

Given a number written in decimal form, how do you determine if it is a repeating decimal?

Extension 14.4

Warm Up

For use before Extension 14.4

Determine if the decimal is *repeating* or *terminating.*

1. 1.222

2. $0.\overline{122}$

3. $23.\overline{546576}$

4. 43.76676

5. $2.\overline{4439}$

6. $0.3\overline{4}$

Name ______________________________ Date __________

Extension 14.4 Practice

Write the decimal as a fraction or mixed number.

1. $0.\overline{2}$

2. $-0.\overline{7}$

3. $-2.\overline{3}$

4. $8.\overline{7}$

5. $-10.\overline{5}$

6. $24.\overline{8}$

7. $0.5\overline{7}$

8. $-1.4\overline{5}$

9. $-3.8\overline{6}$

10. $-0.\overline{32}$

11. $6.\overline{13}$

12. $7.\overline{90}$

Activity 14.5

Start Thinking!

For use before Activity 14.5

Complete the statement:

The Pythagorean Theorem states that if a triangle with legs *a* and *b* and hypotenuse *c* is a right triangle, then __________________.

Give an example of three side lengths of a right triangle.

Can those three side lengths form a triangle that is not a right triangle?

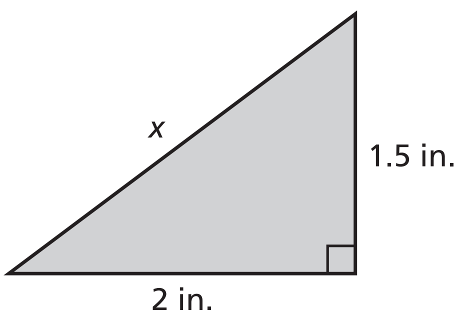
Activity 14.5

Warm Up

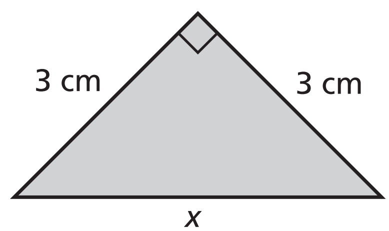
For use before Activity 14.5

Find the missing side length.

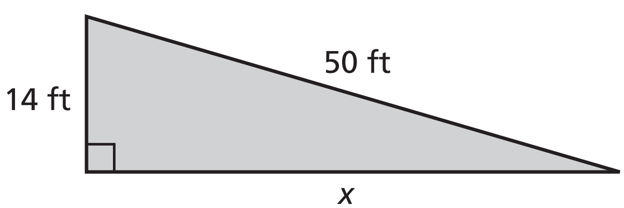
1.



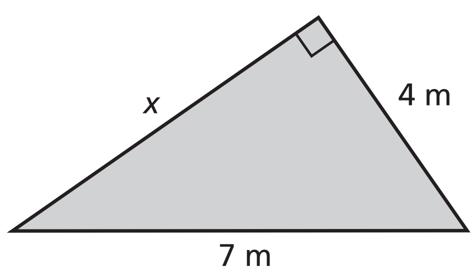
2.



3.



4.



Lesson 14.5

Start Thinking!

For use before Lesson 14.5

Write a word problem that can be solved using the Pythagorean Theorem. Be sure to include a sketch of the situation.

Lesson 14.5

Warm Up

For use before Lesson 14.5

Write the converse of the true statement. Determine whether the converse is true or false. If it is true, justify your reasoning. If it is false, give a counterexample.

1. If a is a negative number, then $|a| = -a$.

2. If one line is vertical and another line is horizontal, then the two lines are perpendicular.

3. If a is a negative number, then a^2 is a positive number.

4. If a is an odd number, then $a - 1$ is an even number.

5. If the side lengths of a triangle are 3, 4, and 5, then the triangle is a right triangle.

6. If a line is given by the equation $y = 2x - 3$, then the y-intercept of the graph is -3.

Name____ Date__________

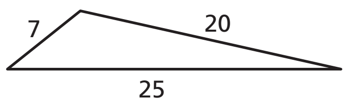
14.5 Practice A

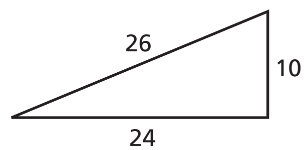
Write the converse of the true statement. Determine whether the converse is true or false. If it is true, justify your reasoning. If it is false, give a counterexample.

1. If a is an odd number, then $2a$ is an even number.

2. If a is negative, then $\frac{1}{a}$ is negative.

Tell whether the triangle with the given side lengths is a right triangle.

3. 

4. 

Find the distance between the two points.

5. $(2, -4), (3, -1)$ **6.** $(3, 2), (7, 5)$ **7.** $(-9, -2), (-7, 5)$

8. The side of the clip on a clip board appears to be a right triangle. The leg lengths are 2 millimeters and 2.1 millimeters and the hypotenuse is 2.9 millimeters. Is the side of the clip a right triangle?

Tell whether a triangle with the given side lengths is a right triangle.

9. 18, 80, 82 **10.** $\sqrt{28}$, 63, 65 **11.** 2, $\sqrt{96}$, 10

12. You are standing 6 feet away from the stage and your friend is standing 7 feet away from the stage.

a. You are standing on a platform, which places your eyes at 6.5 feet. What is the distance from your eyes to the stage?

b. Your friend's eyes are at 5 feet. What is the distance from your friend's eyes to the stage?

c. Do you or your friend have a closer visual?

13. On the Junior League baseball field, you run 60 feet to first base and then 60 feet to second base. You are out at second base and then run directly along the diagonal to home plate. Find the total distance that you ran. Round your answer to the nearest tenth.

Name ____ Date __________

14.5 Practice B

Tell whether the triangle with the given side lengths is a right triangle.

1. 11 in., 60 in., 61 in.

2. 45 cm, 26 cm, 51 cm

Find the distance between the two points.

3. $(5, 5), (8, 7)$

4. $(-6, 2), (3, -2)$

5. $(10, -3), (-1, -8)$

6. Describe and correct the error in finding the distance between the points $(-5, -2)$ and $(-1, 4)$.

$$\begin{aligned} d &= \sqrt{(-5-1)^2 + (-2-4)^2} \\ &= \sqrt{36+36} \\ &= \sqrt{72} \end{aligned}$$

Tell whether a triangle with the given side lengths is a right triangle.

7. $9, \sqrt{54}, 8$

8. $\sqrt{704}, 27, 5$

9. 88, 103, 137

10. Your teacher gives you and your friend two different sets of points and wants both of you to find the distance between the two points.

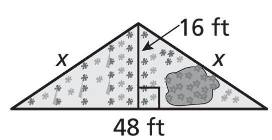
a. Your two points are $(5, 9)$ and $(2, 1)$. Find the distance between the two points.

b. Your friend's two points are $(-5, -9)$ and $(-2, -1)$. Find the distance between the two points.

c. Do you and your friend obtain the same answer? If possible, explain why.

d. Give another example of this situation where the coordinates in the first set of points are a mixture of positive and negative values, and the second set of points have opposite values. Show that the distance is the same.

11. You are creating a flower garden in the triangular shape shown. You purchase edging to go around the flower garden. The edging costs $1.50 per foot. What is the cost of the edging? Round your lengths to the nearest whole number.

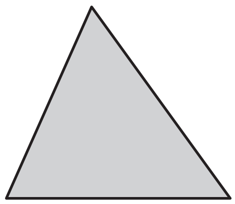


Name____ Date__________

14.5 Enrichment and Extension

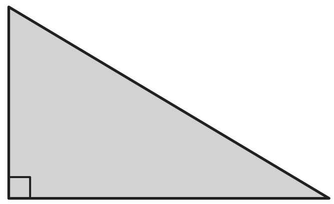
Classifying Triangles

Triangles can be classified as acute, right, or obtuse based on their angle measures.



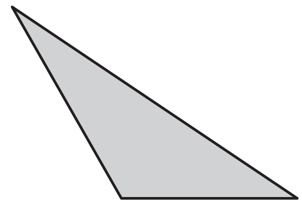
Acute triangle

3 acute angles



Right triangle

1 right angle and
2 acute angles



Obtuse triangle

1 obtuse angle and
2 acute angles

In a triangle where c is the longest side length and a and b are the two other side lengths:

If $a^2 + b^2 > c^2$, then the triangle is acute.

If $a^2 + b^2 = c^2$, then the triangle is right.

If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Determine whether a triangle with the given side lengths is *acute*, *right*, or *obtuse*.

1. 9, 12, and 15

2. 10, 9, and 15

3. 10, 20, and 12

4. 15, 36, and 39

5. 7, 6, and 5

6. 26, 24, and 12

7. 8, 8, and 15

8. 45, 108, and 117

Use the distance formula to tell whether the three points form an *acute*, *right*, or *obtuse* triangle.

9. $(0, 0), (-4, 5), (2, 2)$

10. $(-2, 4), (6, 0), (-5, -2)$

11. $(0, 2), (5, 1), (1, -1)$

12. $(-7, 2), (0, 1), (-4, 4)$

Name ____ Date __________

14.5 Puzzle Time

What Comes After A Seahorse?

Write the letter of each answer in the box containing the exercise number.

Tell whether a triangle with the given side lengths is a right triangle.

1. 8 in., 9 in., 12 in.

R. yes **S.** no

2. 10 cm, 24 cm, 26 cm

A. yes **B.** no

3. 9 mm, 16 mm, 18 mm

N. yes **O.** no

Find the distance between the two points.

4. $(0, 0), (6, 8)$

D. 10 **E.** $\sqrt{300}$

5. $(2, 3), (-3, -2)$

Q. 5 **R.** $\sqrt{50}$

6. $(4, -7), (5, -1)$

D. 37 **E.** $\sqrt{37}$

7. During math class, your friend sits in the second row, third seat, which can be represented by the point $(2, 3)$. You sit in the fifth row, first seat, which can be represented by the point $(5, 1)$. What is the distance between your seats?

G. 13 **H.** $\sqrt{13}$

2		4		7	3	5	1	6
			–					

Name____ Date___________

Chapter 14

Technology Connection

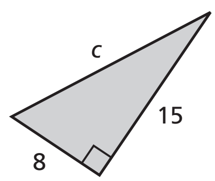
For use after Section 14.3

Using the Pythagorean Theorem

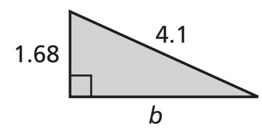
The Pythagorean Theorem is one of the most famous theorems in mathematics partly because it can be used to solve so many types of problems. Although the theorem only applies to right triangles, right triangles themselves can be found within all rectangles and squares. Because of this, knowing how to find any side of a right triangle using the Pythagorean Theorem and a scientific calculator is an extremely useful skill.

EXAMPLE Use a scientific calculator to solve each right triangle.

a.



b.



SOLUTION

a. Because c is the hypotenuse, $c^2 = a^2 + b^2$. To solve for c, enter the following keystrokes on your calculator.

$$8\,\boxed{x^2}\,\boxed{+}\,15\,\boxed{x^2}\,\boxed{=}\,\boxed{\sqrt{\ }}, \text{ or, } \boxed{\sqrt{\ }}\,\boxed{(}\,8\,\boxed{x^2}\,\boxed{+}\,15\,\boxed{x^2}\,\boxed{)}\,\boxed{=}$$

The display should show 17.

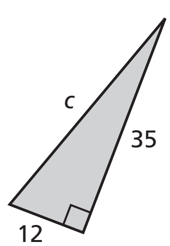
b. Because b is a leg, you can modify the Pythagorean Theorem and solve for b and find that $b^2 = c^2 - a^2$. Enter the following keystrokes on your calculator.

$$4.1\,\boxed{x^2}\,\boxed{-}\,1.68\,\boxed{x^2}\,\boxed{=}\,\boxed{\sqrt{\ }}, \text{ or, } \boxed{\sqrt{\ }}\,\boxed{(}\,4.1\,\boxed{x^2}\,\boxed{-}\,1.68\,\boxed{x^2}\,\boxed{)}\,\boxed{=}$$

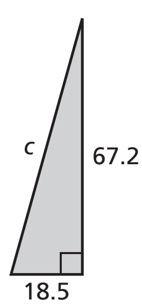
The display should show 3.74.

Use a scientific calculator to solve the right triangle.

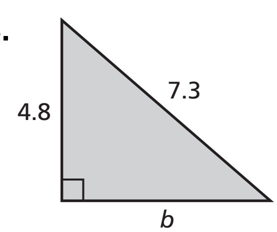
1.



2.



3.



4.